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Short Communication

A note on the proportionate flow shop with a bottleneck machine

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Abstract

Choi, B.-C., Yoon, S.-H., Chung, S.-J., 2007. Minimizing maximum completion time in a proportionate flow shop with one machine of different speed. European Journal of Operational Research 176, 964–974 consider the proportionate flow shop with a slow bottleneck machine and propose the SLDR heuristic for it. Choi et al. (2007) derive a data-dependent worst-case ratio bound for the SLDR heuristic which is then bounded by two. In this note, we show that the tight worst-case ratio bound of the SLDR heuristic is 3/2.

Keywords: Flow shop sequencing; Proportionate flow shop; Makespan; Approximation algorithms

1. Introduction

Choi et al. (2007) consider the proportionate flow shop with a slow bottleneck machine (other than the first or the last machine) in which each job has the same processing time on all machines (except on the slow bottleneck machine). Formally, there are n jobs j, $j = 1, \dots, n$ all of them available at time zero; each job must be processed non-preemptively and sequentially on m continuously available machines i, $i = 1, \dots, m$. Each machine i can process at most one job at a time and any operation of a job cannot start until the preceding operation of that job has been completed. Let $[j]$ denote the job sequenced in the j position in the overall schedule and let $C_{i[j]}$ denote its completion time on machine i. The objective is to determine a job sequence S on all machines so that the maximum job completion time (makespan) $C^S_{\max} = C^S_{m[n]}$ is minimized. Choi et al. (2007) state that permutation schedules (in which the jobs are processed in the same order on all machines) are dominant. Let p_j denote the units of processing of job j and let p_{ij} be the actual processing time of job j on machine i; let k, $1 < k < m$ be the slow bottleneck machine. Then, $p_{ij} = p_j$ for all $i \neq k$ and $p_{kj} = p_j S_k$ with $S_k > 1$. The SLDR heuristic sequences the jobs in the shortest processing time (SPT) order when $k \geqslant \frac{m}{2}$ and in the longest processing time (LPT) order otherwise. The tight worst-case ratio bound of 3/2 for the SLDR heuristic is derived in the next section.

2. A worst-case ratio bound for the SLDR heuristic

We consider the $k \geqslant \frac{m}{2}$ case in which the SLDR heuristic supplies the SPT sequence. The $k < \frac{m}{2}$ can be handled analogously by invoking the time reversibility principle of flow shop makespan problems.

Proposition 1. $\frac{C^{\mathrm{SPT}}_{\max}}{C^*_{\max}} \leqslant \frac{3}{2}$ *for the* $k \geqslant \frac{m}{2}$ *case and this bound is tight where* $C^{\mathrm{SPT}}_{\max}$, $C^*_{\max}$ *denote the makespan of the SPT sequence and the optimal makespan, respectively.*

Proof. According to Smith et al. (1975), the SPT sequence is optimal for the k-machine flow shop problem comprised of machines 1 through k since the last machine is the slow bottleneck machine in that problem; (actually Smith et al. (1975) proved this result for the more general ordered flow shop problem in which if $p_{ij} \leqslant p_{ik}$ for any two jobs j, k on any machine i, then $p_{ij} \leqslant p_{ik}$ on all machines i, $i = 1, \dots, m$, and if $p_{ij} \leqslant p_{kj}$ for any job j on any two machines i, k, then

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$p_{ij} \leqslant p_{kj}$ for all jobs j, $j = 1, \ldots, n$). Consequently:

$$C^{SPT}_{k[n]} \leqslant C^*_{max}, \tag{1}$$

because the optimal makespan of the k-machine problem comprised of machines 1 through k is a lower bound on C^*_{max}. As in Choi et al. (2007):

$$C^{SPT}_{max} = C^{SPT}_{k[n]} + (m-k)p_{[n]}. \tag{2}$$

Also,

$$(m-k)p_{[n]} \leqslant \frac{m}{2}p_{[n]} \leqslant \frac{1}{2}[(m-1)p_{[n]} + p_{k[n]}] \leqslant \frac{1}{2}C^*_{max}. \tag{3}$$

The last inequality in Eq. (3) is a direct consequence of the observation that the total processing time of any job is a lower bound on C^*_{max}. The combination of Eqs. (1)–(3) yields the desirable bound of $\frac{C^{SPT}_{max}}{C^*_{max}} \leqslant \frac{3}{2}$. □

In order to prove tightness, we consider a problem with $2k+1$ jobs, $2k+1$ machines and with machine $k+1$ as the bottleneck machine. Let $p_1 = \cdots = p_{2k} = 1$, $p_{2k+1} = M$ and $S_{k+1} = M$ where M is a large positive number. The SPT rule yields the job sequence $1, \ldots, 2+1$ with $C^{SPT}_{max} = 3Mk + M^2 + k$. The optimal job sequence is $1, \ldots, k$, $2k+1$, $k+1, \ldots, 2k$ with $C^*_{max} = 2Mk + M^2 + 2k$. Consequently, $\frac{C^{SPT}_{max}}{C^*_{max}} = \frac{3Mk+M^2+k}{2Mk+M^2+2k} = \frac{3+\frac{M}{k}+\frac{1}{M}}{2+\frac{M}{k}+\frac{2}{M}} \to \frac{3+\frac{1}{M}}{2+\frac{2}{M}}$ as $k \to \infty$; furthermore, M can be selected sufficiently large so that $\frac{C^{SPT}_{max}}{C^*_{max}}$ is arbitrarily close to 3/2.

By invoking the time reversibility principle of flow shop makespan problems, we state the following corollary without proof:

Corollary 1. *$\frac{C^{LPT}_{max}}{C^*_{max}} \leqslant \frac{3}{2}$ for the $k < \frac{m}{2}$ case and this bound is tight where C^{LPT}_{max}, C^*_{max} denote the makespan of the LPT sequence and the optimal makespan, respectively.*

Proposition 1 and Corollary 1 can be combined into the main result of this note which is stated next without proof.

Theorem 1. *$\frac{C^{SLDR}_{max}}{C^*_{max}} \leqslant \frac{3}{2}$ and this bound is tight where C^{SLDR}_{max}, C^*_{max} denote the makespan of the SLDR heuristic and the optimal makespan, respectively.*

3. Concluding remarks

Choi et al. (2007) derive the upper bound of two for the worst-case ratio bound of the SLDR heuristic by bounding a data-dependent bound which according to Choi et al. (2007) is tight; however, Choi et al. (2007) do not measure the maximum deviation of the SLDR solution from the optimal solution because the heuristic makespan and the optimal makespan are equal in the problem used by Choi et al. (2007) to prove tightness. Our findings show that the actual tight worst-case ratio bound for the SLDR heuristic is 3/2.

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