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Short Communication

A faster fully polynomial approximation scheme for the single-machine total tardiness problem

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Abstract

Lawler [E.L. Lawler, A fully polynomial approximation scheme for the total tardiness problem, Operations Research Letters 1 (1982) 207–208] proposed a fully polynomial approximation scheme for the single-machine total tardiness problem which runs in $O\left(\frac{n^7}{\varepsilon}\right)$ time (where n is the number of jobs and ε is the desired level of approximation). A faster fully polynomial approximation scheme running in $O(n^5 \log n + \frac{n^5}{\varepsilon})$ time is presented in this note by applying an alternative rounding scheme in conjunction with implementing Kovalyov's [M.Y. Kovalyov, Improving the complexities of approximation algorithms for optimization problems, Operations Research Letters 17 (1995) 85–87] bound improvement procedure.

Keywords: Single-machine sequencing; Total tardiness; Fully polynomial approximation

1. Introduction

The single-machine total (average) tardiness problem $1//\overline{T}$ is defined as follows: There are n jobs available at time zero; job j has a processing time p_j and a due date d_j. The tardiness of job j is defined as $T_j = \max(0, C_j - d_j)$ where C_j is the completion time of job j in a given sequence. The objective is to determine a job sequence such that the total tardiness $\sum_{j=1}^{n} T_j$ is minimized. The $1//\overline{T}$ problem is ordinary NP-hard [3]. Lawler [5] developed a decomposition-based optimal pseudo-polynomial algorithm for the $1//\overline{T}$ problem (to be called the OPP algorithm from now on) which runs in $O(n^4P)$ time (where $P = \sum_{j=1}^{n} p_j$). Other decomposition-based optimal algorithms for the $1//\overline{T}$ problem were developed by Potts and Van Wassenhove [7] (under the assumption that the longest job is completed as late as possible in an optimal sequence when it cannot be completed on time) and by Chang et al. [1] (under the assumption that the longest job is completed as early as possible in an optimal sequence). All of these decomposition-based optimal algorithms have the same worst-case running time as the OPP algorithm. Szwarc [8] presents a unified framework for the decomposition theorem of the $1//\overline{T}$ problem.

The complexity of the $1//\overline{T}$ problem justified the development of heuristics. According to Della Croce et al. [2], the worst-case bound for most of these heuristics is arbitrarily bad since it is a function of n. This is true even for decomposition heuristics which are heuristic implementations of the decomposition property of the $1//\overline{T}$ problem.

A fully polynomial approximation scheme for the $1//\overline{T}$ problem was developed by Lawler [6] by modifying his OPP algorithm. It supplies a sequence with total tardiness T in $O\left(\frac{n^7}{\varepsilon}\right)$ time such that $T - T^* \leqslant \varepsilon T^*$ where T^* is the optimal solution and ε is the desired level of approximation. This is accomplished by applying the OPP algorithm to a $1//\overline{T}$ problem with rounded rescaled processing times and non-rounded rescaled due dates. A faster fully polynomial approximation scheme running in $O(n^5 \log n + \frac{n^5}{\varepsilon})$ time can be developed by applying the OPP algorithm to a $1//\overline{T}$

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problem with non-rounded rescaled processing times and rounded rescaled due dates in conjunction with implementing Kovalyov's [4] bound improvement procedure.

2. The proposed fully polynomial approximation scheme

It is well known that

$$T_{\max} \leqslant T^* \leqslant nT_{\max} \tag{1}$$

where $T_{\max} = \max\{T_j\}$ for the earliest due date (EDD)sequence, that is, $T_{\max}$ is a lower bound (LB) and $nT_{\max}$ is an upper bound (UB) on T^*. Lawler [6] points out that instead of running his OPP algorithm in the $[0, P]$ interval, it suffices to run it in the $[0, \mathrm{UB}]$ interval, that is the $[0, nT_{\max}]$ interval when $\mathrm{UB} = nT_{\max}$, resulting in a $\mathrm{O}(n^4\mathrm{UB}) = \mathrm{O}(n^5T_{\max})$ running time for OPP.

Let us replace the due dates d_j with the rescaled due dates $\delta_j = \left\lceil \frac{d_j}{K} \right\rceil$ where K is a scale factor proportional to the desired level of approximation ε (the function $\lceil\,\rceil$ returns the smallest integer greater or equal than its argument). The processing times p_j are also replaced by the new processing times $\frac{p_j}{K}$ (with no rounding).

Let S_A be an optimal sequence for the $\left(\frac{p_j}{K}, \delta_j\right)$ problem and let T_A^*, T_A be the total tardiness of S_A for the $(p_j, K\delta_j)$ and (p_j, d_j) problems, respectively. The inequality $(\delta_j - 1)K < d_j \leqslant \delta_j K$ leads to $C_j - \delta_j \ \ K \leqslant C_j - d_j < C_j - \delta_j K + K$ for $j = 1, \ldots, n$ which in turn leads to

$$T_A^* \leqslant T_A < T_A^* + Kn \tag{2}$$

The inequality $K\delta_j \geqslant d_j$ leads to

$$T_A^* \leqslant T^* \tag{3}$$

because T_A^* and T^* are both optimal quantities for the $(p_j, K\delta_j)$ and (p_j, d_j) problems, respectively. The combination of (2) and (3) leads to

$$T_A \leqslant T^* + Kn \tag{4}$$

If $K = \frac{\varepsilon \mathrm{LB}}{n} = \frac{\varepsilon T_{\max}}{n}$ is substituted in inequality (4), then the combination of (1) and (4) yields $T_A - T^* \leqslant \varepsilon \mathrm{LB} = \varepsilon T_{\max} \leqslant \varepsilon T^*$, the desired approximation. Furthermore, the $\mathrm{O}(\frac{n^4\mathrm{UB}}{K})$ time bound of the OPP algorithm for solving the $\left(\frac{p_j}{K}, \delta_j\right)$ problem becomes $\mathrm{O}\left(\frac{n^5\mathrm{UB}}{\varepsilon\mathrm{LB}}\right) = \mathrm{O}\left(\frac{n^6}{\varepsilon}\right)$ for the selected K value and for $\mathrm{LB} = T_{\max}$ and $\mathrm{UB} = nT_{\max}$, respectively.

Kovalyov [4] proposed a bound improvement procedure which when applied to the $\mathrm{LB} = T_{\max}$ and $\mathrm{UB} = nT_{\max}$ values (assuming that $n > 3$) with our rounding approximation scheme (with $\varepsilon = 1$) embedded in it will find a number F^0 such that $F^0 \leqslant T^* \leqslant 3F^0$ in $\mathrm{O}(n^5 \log n)$ time. These improved bounds can then be used in place of the $\mathrm{LB} = T_{\max}$ and $\mathrm{UB} = nT_{\max}$ values in our rounding approximation scheme to yield the desired approximation of $T_A - T^* \leqslant \varepsilon T^*$ in $\mathrm{O}(n^5 \log n + \frac{n^5}{\varepsilon})$ overall time.

In summary, a fully polynomial approximation scheme running in $\mathrm{O}\left(n^5 \log n + \frac{n^5}{\varepsilon}\right)$ time can be developed for the ordinary NP-hard $1//\overline{T}$ problem (whenever $T_{\max} > 0$ for the EDD sequence) by first computing $K = \frac{\varepsilon T_{max}}{n}$, then embedding the OPP algorithm with the non-rounded rescaled processing times $\frac{p_j}{K}$ and the rounded rescaled due dates $\delta_j = \left\lceil \frac{d_j}{K} \right\rceil$ in Kovalyov's [4] bound improvement procedure, and finally running the OPP algorithm again utilizing the improved bounds obtained from Kovalyov's [4] procedure. If $T_{\max} = 0$ for the EDD sequence, then the $1//\overline{T}$ problem is solved optimally in $\mathrm{O}(n \log n)$ time by implementing the EDD sequence.

3. Conclusions

A fully polynomial approximation scheme running in $\mathrm{O}\left(n^5 \log n + \frac{n^5}{\varepsilon}\right)$ time was developed for the $1//\overline{T}$ problem. The proposed algorithm runs faster than the original fully polynomial approximation scheme developed by Lawler [6]. The computational savings stem from rounding the rescaled due dates (instead of rounding the rescaled processing times) and from applying Kovalyov's [4] bound improvement procedure.

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